Exercise 10

Use $i = (0,1)$ and $y = (y,0)$ to verify that $-(iy) = (-i)y$. Thus show that the additive inverse of a complex number $z = x + iy$ can be written $-z = -x - iy$ without ambiguity.

Solution

Evaluate the left side.

$$\begin{aligned}
-(iy) &= -[(0,1)(y,0)] \\
&= -[(0-0, y+0)] \\
&= -(0,y)
\end{aligned}$$

Evaluate the right side.

$$\begin{aligned}
(-i)y &= (0,-1)(y,0) \\
&= (0-0, -y+0) \\
&= (0,-y) \\
&= -(0,y)
\end{aligned}$$

Consequently, it doesn't matter whether $-z$ is written as $-x + (-i)y$ or $-x - (iy)$ or $-x - iy$.